

On the Quantum Geometry of String Theory

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The IKKT or IIB matrix model has been proposed as a non-perturbative definition of type IIB superstring theories. It has the attractive feature that space-time appears dynamically. It is possible that lower dimensional universes dominate the theory, therefore providing a dynamical solution to the reduction of space-time dimensionality. We summarize recent works that show the central role of the phase of the fermion determinant in the possible realization of such a scenario.

One of the most important problems in string theory is the emergence of four dimensional space-time in low energy physics. Compactification schemes have been the most popular approach of the issue, but the price to pay is loss of predictability. One possible scenario is that 4d space-time appears dynamically from an originally higher dimensional theory, usually $10D$ or $11D$. A promising model that might realize this scenario is the IKKT matrix model [1], which is supposed to define type IIB superstrings in the large N limit non-perturbatively. One can view this model as a “lattice string theory” and study it using standard Monte Carlo techniques. Formally it is the zero-volume limit of $D = 10$, $\mathcal{N} = 1$ super Yang-Mills theory. The partition function of the model (and its generalizations to $D = 4, 6$) can be written as

$$Z = \int dA e^{-S_b} Z_f[A], \quad (1)$$

where A_μ ($\mu = 1, \dots, D$) are D bosonic $N \times N$ traceless hermitian matrices, and $S_b = -\frac{1}{4g^2} \text{Tr}([A_\mu, A_\nu]^2)$ is the bosonic part of the action. The factor $Z_f[A]$ represents the quantity obtained by integration over the fermionic matrices, and its explicit form is given for example in Ref. [2]. Space-time appears dynamically (the eigenvalues of the bosonic matrices A_μ represent

space-time points) and if the dominant configurations have d extended dimensions and $D - d$ small ones then we obtain essentially a d dimensional space-time *dynamically*. This requires in particular the spontaneous symmetry breaking (SSB) of the manifest $SO(D)$ invariance of the model. We stress here that the parameter g appearing in eq. (1) is not a coupling constant but a scale which can be absorbed by a redefinition of the fields. One hopes to arrive at $d = 4$ in the $10D$ IKKT model.

Such a scenario has not been verified. There are severe technical problems in studying the $10D$ IKKT model by computer simulations, related to the complex action problem. The study of simpler models has shed some light on the possible mechanisms that could realize the above mentioned scenario. We have learned that SSB is not realized in the absence of the rapidly oscillating phase Γ of the (complex) fermionic partition function $Z_f[A]$. In Ref. [3] it has been shown that when fermions are absent (“bosonic model”) SSB does not occur. The study of a low energy approximation of the $10D$ and $6D$ models, where Γ is set to zero by hand, did not reveal any indication for SSB [4]. When the effect of Γ is infinitely enhanced in the $10D$ case by appropriately deforming the original model, it turned out that

space time is $2 < d \leq 8$, i.e. the SSB scenario is realized [5]. Recently it has been shown in exactly solvable matrix models that SSB of $SO(D)$ occurs precisely due to the phase of the fermion determinant [6].

The purpose of this work is twofold: First to show that in the $4D$ model SSB does not occur [7]. The $4D$ model has the property that $Z_f[A]$ is real positive, $\Gamma = 0$, and it can be studied using ordinary Monte Carlo methods. Therefore the result supports the importance of the phase in the realization of SSB. The authors of Ref. [8] have raised the possibility that one dimensional structures dominate eq. (1). We propose different, more physical probes of space-time dimensionality that need to be used in $D = 4$. Due to the power tail of eigenvalue distributions [9], the observable $T_{\mu\nu} = \frac{1}{N} \text{Tr}(A_\mu A_\nu)$ used as a probe in Ref. [8] is ill defined (note that $\langle \text{Tr} A^2 \rangle$ is divergent). This does not happen in higher dimensions where one expects all probes discussed here to give identical answers. Of course in any dimension $\langle \text{Tr} A^m \rangle$ diverges for large enough m , therefore it is important to understand and resolve this puzzle in a convincing way.

Second we show that it is possible to attack the problem of SSB in higher dimensions despite the complex action problem. We propose a method that allows us to compute the $T_{\mu\nu}$ eigenvalue distribution in $6D$ for large N . The method does not suffer from the complex action problem and it might be applicable to other models as well. The resulting distribution is qualitatively different from the case $\Gamma = 0$, due to the presence of the phase. A double peak structure appears for large enough N , raising the possibility that small eigenvalues dominate for some dimensions while large ones dominate for others, which would mean that SSB occurs. A more involved study, possible however, is needed in order to resolve the issue.

A natural probe of space-time dimensionality is its “moment of inertia” tensor $T_{\mu\nu} = \frac{1}{N} \text{Tr}(A_\mu A_\nu)$. Its D eigenvalues $\lambda_1 > \lambda_2 > \dots > \lambda_D > 0$ represent the principal moments of inertia. In $4D$, however, its largest eigenvalue is known to diverge [7], in agreement with the divergence of $\langle \text{Tr} A^2 \rangle$ [9]. In this case it is possible

to modify its definition to

$$T_{\mu\nu}^{(\text{new})} = \left\langle \sum_{i < j} \frac{2(x_{i\mu} - x_{j\mu})(x_{i\nu} - x_{j\nu})}{N(N-1)\sqrt{(x_i - x_j)^2}} \right\rangle.$$

$x_{i\mu}$ are the space-time points defined to be the diagonal elements of the bosonic matrices A_μ when they are transformed into a form as close to simultaneously diagonalized as possible [3,2].

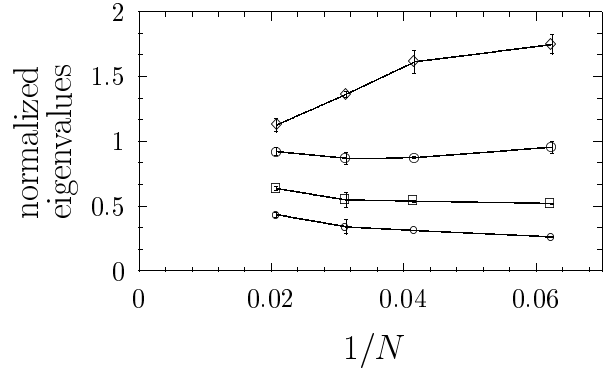


Fig. 1: The eigenvalues $\lambda_i^{(\text{new})}/(\sqrt{g}N^{1/4})$.

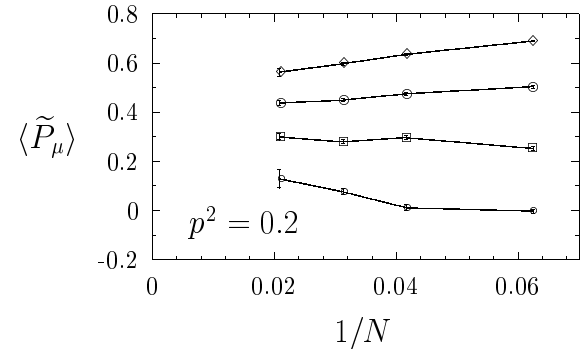


Fig. 2: The four Polyakov lines $\langle \tilde{P}_\mu(p) \rangle$ ($\mu = 1, \dots, 4$) at $p^2 = 0.2$.

Since the distance between space-time points behaves asymptotically as $\rho(r) \sim r^{-3}$, this quantity is well defined. In Ref. [7] we found that for increasing N , the eigenvalues $\lambda_1^{(\text{new})} \dots \lambda_4^{(\text{new})}$ show no trend for SSB. This can be seen in Fig. 1 where all eigenvalues seem to approach the same value with increasing N . The same result is obtained from the study of Polyakov lines

$P(\vec{p}) = \frac{1}{N} \text{Tr} \exp(i p_\mu A_\mu)$. They are essentially the Fourier transform of $\rho(r)$ and their characteristic fall off is a measure of the extent of space-time [2]. Since Wilson loops in general have been interpreted as string creation operators they correspond to observables in string theory. Defining the Polyakov lines on field configurations \tilde{A}_μ that diagonalize $T_{\mu\nu}$ as $\tilde{P}_\mu(p) = \frac{1}{N} \text{Tr} \exp(i p \tilde{A}_\mu)$ we can study if SSB occurs. In Fig. 2 we see that all $\tilde{P}_\mu(p)$ seem to converge to the same universal function, so no trend for SSB is observed [7]. This behavior is typical for all values of p^2 that we measured.

For the $6D$ partition function eq. (1) $\Gamma \neq 0$ and the model suffers from the complex action problem. The partition function can be written as $Z = \int dA e^{-S_0} e^{i\Gamma}$, where $Z_f[A] = \exp(\Gamma_R + i\Gamma)$, $\Gamma_R, \Gamma \in \mathbb{R}$ and $S_0 = S_b - \Gamma_R$. The rapidly fluctuating $e^{i\Gamma}$ term makes the calculation of expectation values exponentially hard with increasing system size. Furthermore, the model defined by the partition function $Z_0 = \int dA e^{-S_0}$ used in the Monte Carlo simulation visits very rarely the part of configuration space that dominates in the full model (overlap problem). It is possible though to overcome these technical obstacles. One can study the distribution of $\tilde{\lambda}_i = \frac{\lambda_i}{\langle \lambda_i \rangle_0}$ defined by $\rho_i(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle$, where $\langle \dots \rangle_0$ are expectation values with respect to Z_0 . In Ref. [10] we observed

$$\rho_i(x) = \frac{1}{C} \rho_i^{(0)}(x) e^{N^2 \Phi_i(x)} \quad (2)$$

where $\rho_i^{(0)}(x)$ is the $\tilde{\lambda}_i$ distribution in the Z_0 ensemble, $\Phi_i(x) = \frac{1}{N^2} \log \langle \cos \Gamma \rangle_{0, \tilde{\lambda}_i=x}$ and $C = \langle \cos \Gamma \rangle_0$ is a normalization constant. $\langle \dots \rangle_{0, \tilde{\lambda}_i=x}$ are expectation values in the $Z_{0, \tilde{\lambda}_i=x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_i)$ ensemble. It turns out that the function $\Phi_i(x)$ converges to an N -independent scaling function for quite small N (≤ 20). Combining this observation with the factorization property (2), we see that $\rho_i(x)$ can be computed for much larger N than $\Phi_i(x)$ since the remaining terms in eq. (2) do not suffer from the complex action problem.

In Ref. [10] we simulated the low energy version of the $6D$ IKKT model [1] for N up to 128. We computed the scaling function $\Phi_i(x)$, $i = 4, 5$

using $N \leq 20$. We found that $\rho_i(x)$, $i = 4, 5$ have two peaks at $x = x_<$ and $x = x_>$ ($x_< < 1$ and $x_> > 1$), which is qualitatively different from $\rho_i^{(0)}(x)$, with peaks around 1. It is important to determine the peak which dominates the large- N limit. In Ref. [10] we found that the data are consistent with a scenario where $x_<$ dominates $\rho_5(x)$ whereas $x_>$ dominates $\rho_4(x)$. This behavior can be expected since the qualitative difference in the behaviors of $\rho_5(x)$ and $\rho_4(x)$ is mainly due to $\rho_5^{(0)}(x)$ and $\rho_4^{(0)}(x)$. In the branched polymer description of the low energy effective theory of the IIB matrix model [1], $\rho_4^{(0)}(x)$ is expected to be much more suppressed in the small x regime than $\rho_5^{(0)}(x)$. The method described above is quite general and can be applied to other physical systems that suffer from the complex action problem. Such a system is finite density QCD, and we are currently investigating the related random matrix theory with very encouraging results [11].

REFERENCES

1. N. Ishibashi et al., Nucl. Phys. **B498** (1997) 467; H. Aoki et al., Prog. Theor. Phys. **99** (1998) 713.
2. J. Ambjørn et al., JHEP **0007** (2000) 013.
3. T. Hotta et al., Nucl. Phys. **B545** (1999) 543.
4. J. Ambjørn et al., JHEP **0007** (2000) 011.
5. J. Nishimura and G. Vernizzi, JHEP **0004** (2000) 015; Phys. Rev. Lett. **85** (2000) 4664.
6. J. Nishimura, hep-th/0108070.
7. J. Ambjørn et al., hep-th/0104260.
8. P. Bialas et al., Nucl. Phys. **B592** (2001) 391; Z. Burda et al., Nucl. Phys. **B602** (2001) 399.
9. W. Krauth et al, Phys. Lett. **B431** (1998) 31; P. Austing and J. F. Wheeler, JHEP **0102**, 028 (2001); JHEP **0104**, 019 (2001).
10. K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041.
11. J. Ambjørn, K. N. Anagnostopoulos, J. Nishimura and J. Verbaarschot, in prep.